Fall 2025	CS5368 Intelligent Systems	Assignment 4 Problem solving

First Name	
Last Name	
Student ID	
Due date	December 1st (before the class for 001 and by the end of the day for D01)
Max grade	50

Please answer the following questions and submit them through Canvas. Be sure to submit it to the Assignment 4 problem-solving link.

Problem 1 [15 pts]: Probabilities

A. [5 pts] Let X, Y, and Z be discrete random variables with the following domains:

- $X \in \{x_1, x_2, x_3\}$(3 values)
- $Y \in \{y_1, y_2, y_3\}$(3 values)
- $Z \in \{z_1, z_2, z_3, z_4\}$(4 values)

How many entries are in the following probability tables, and what is the sum of the values in each table? Write a "?" in the box if there is not enough information given.

Table	Size	Sum
$P(X,Z\|Y)$		
$P(Y\|X,Z)$		
$P(z1\|X)$		
$P(X,z3)$		
$P(X \mid y2, z3)$		

B. [5 pts] State whether each of the following statements is True or False. Justify. No independence assumptions are made.

1. $P(A, B) = P(A|B)\,P(A)$

2. $P(A|B)P(C|B) = P(A, C|B)$

3. $P(B,C) = \Sigma a \in A\; P(B, C|A)$

4. $P(A, B, C, D) = P(C)P(D|C)P(A|C, D)P(B|A, C, D)$

5. $P(C|B, D) = \frac{P(B)P(C|B)P(D|C,B)}{\sum_c P(B)P(c|B)P(D|c,B)}$

C. [5 pts] For the following questions, you will be given a set of probability tables and a set of conditional independence assumptions. Given these tables and independence assumptions, write an expression for the requested probability tables. Keep in mind that your expressions cannot contain any probabilities other than the given probability tables. If it is not possible, write "Not possible."

(i) Using probability tables P(A), P(A | C), P(B | C), P(C | A, B) and no conditional independence assumptions, write an expression to calculate the table P(A, B | C).

P(A, B | C) = ___________________________

(ii) Using probability tables P(A), P(A | C), P(B | A), P(C | A, B) and no conditional independence assumptions, write an expression to calculate the table P(B | A, C).

P(B | A, C) = ___________________________

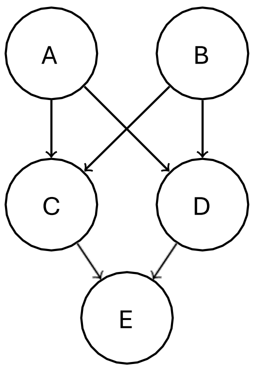
(iii) Using probability tables P(A | B), P(B), P(B | A, C), P(C | A) and conditional independence assumption A ⊥ B, write an expression to calculate the table P(C).

P(C) = ___________________________

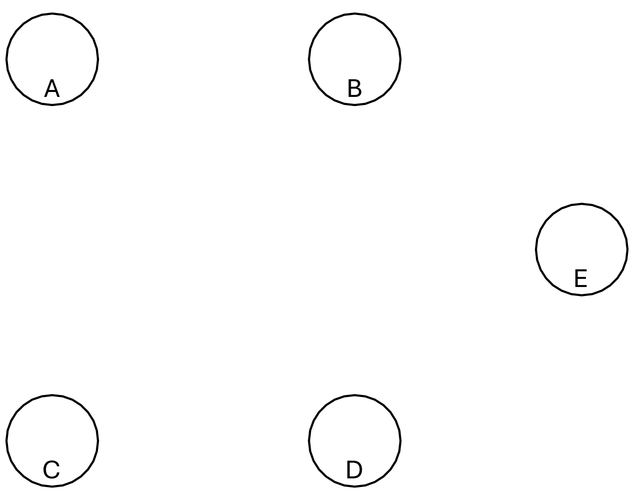
(iv) Using probability tables P(A | B, C), P(B), P(B | A, C), P(C | B, A) and conditional independence assumption A ⊥ B | C, write an expression for P(A, B, C).

Problem 2 [15 pts]: BN representation

A. [2 pts] Write down the joint probability distribution associated with the following Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables associated with this Bayes Net:



B. [2 pts] Draw the Bayes net associated with the following joint distribution:
P (A) · P (B) · P (C|A, B) · P (D|C) · P (E|B, C)



C. [5 pts] Consider a joint distribution over N variables. Let k be the domain size for all of these variables, and let d be the maximum indegree of any node in a Bayes net that encodes this distribution.

(i) What is the space complexity of storing the entire joint distribution? Give an answer of the form O(·).

(ii) Draw an example of a Bayes net over four binary variables such that it takes less space to store the Bayes net than to store the joint distribution.

(iii) Draw an example of a Bayes net over four binary variables such that it takes more space to store the Bayes net than to store the joint distribution.

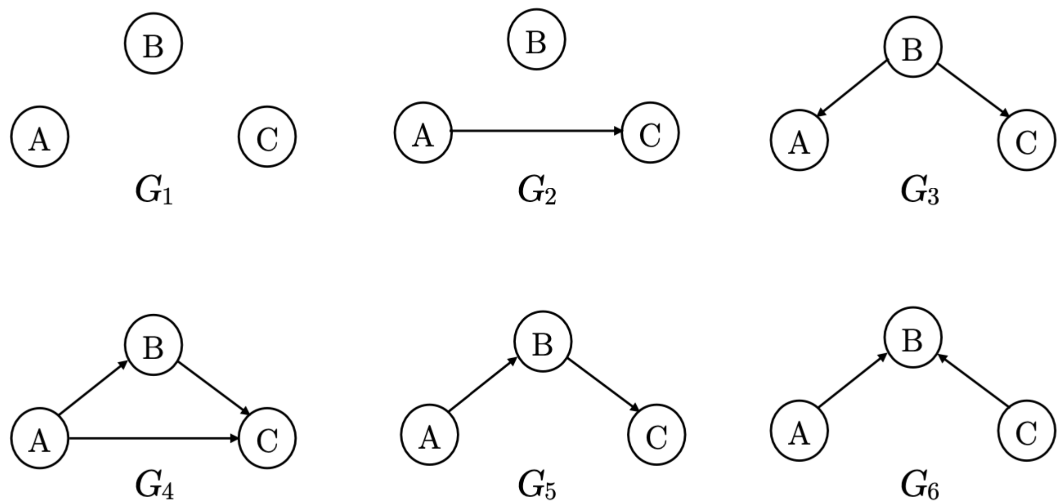
D. [2 pts] What factor can be multiplied with the following factors to form a valid joint distribution? (Write "none" if the given set of factors can't be turned into a joint by the inclusion of exactly one more factor.)

(i) $P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|B, C, D)$

(ii) $P(D) \cdot P(B) \cdot P(C|D, B) \cdot P(E|C, D, A)$

E. [4 pts] Recall that any directed acyclic graph G has an associated family of probability distributions, which consists of all probability distributions that can be represented by a Bayes' net with structure G.

For the following questions, consider the following six directed acyclic graphs:



(i) Assume all we know about the joint distribution $P(A, B, C)$ is that it can be represented by the product $P(A|B, C)P(B|C)P(C)$. Mark each graph for which the associated family of probability distributions is guaranteed to include $P(A, B, C)$.

☐ G_1 ☐ G_2 ☐ G_3

☐ G_4 ☐ G_5 ☐ G_6

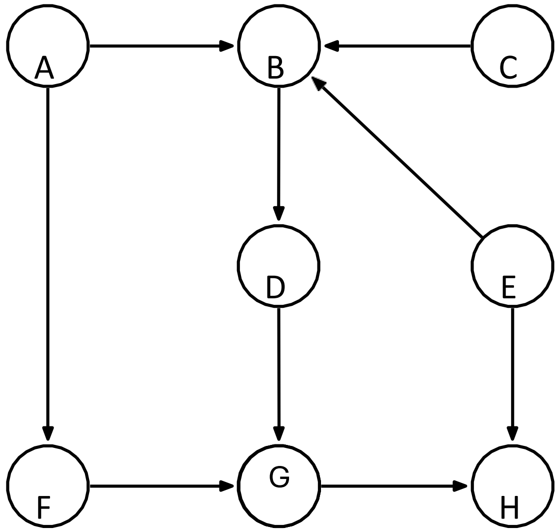
(ii) Now assume all we know about the joint distribution $P(A, B, C)$ is that it can be represented by the product $P(C|B)P(B|A)P(A)$. Mark each graph for which the associated family of probability distributions is guaranteed to include $P(A, B, C)$.

☐ G_1 ☐ G_2 ☐ G_3

☐ G_4 ☐ G_5 ☐ G_6

Problem 3 [10 pts]: BN Independence

Consider the Bayes' net given below.



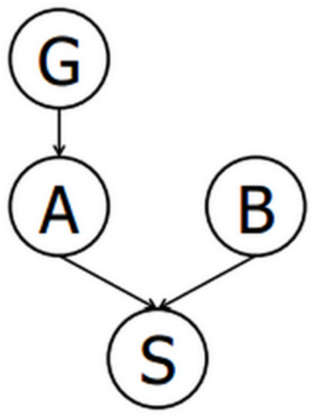
Remember that $X \perp Y$ reads as "X is independent of Y given nothing" and $X \perp Y | Z$ reads as "X is independent of Y given Z". For each expression, indicate whether it is true or false. Explain

1. It is guaranteed that $A \perp B$
2. It is guaranteed that $A \perp C$
3. It is guaranteed that $A \perp D \mid \{B, H\}$
4. It is guaranteed that $A \perp E | F$
5. It is guaranteed that $C \perp H | G$

Problem 4 [10 pts]: BN Inference

a. [7 pts] Consider the following Bayes net to answer the questions below.

G	$P(G)$
g	0.10
$\neg g$	0.90



B	$P(B)$
b	0.40
$\neg b$	0.60

G	A	$P(A \mid G)$
g	a	1.00
g	$\neg a$	0.00
$\neg g$	a	0.10
$\neg g$	$\neg a$	0.90

A	B	S	$P(S \mid A, B)$
a	b	s	1.00
a	b	$\neg s$	0.00
a	$\neg b$	s	0.90
a	$\neg b$	$\neg s$	0.10
$\neg a$	b	s	0.80
$\neg a$	b	$\neg s$	0.20
$\neg a$	$\neg b$	s	0.10
$\neg a$	$\neg b$	$\neg s$	0.90

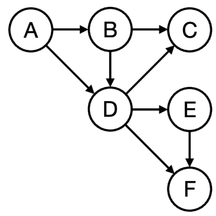
(i) [1 pt] Compute $P(g, a, b, s)$

(ii) [2 pts] What is the probability that a patient has disease A?

(iii) [2 pts] What is the probability that a patient has disease A given that they have symptom S and disease B?

(iv) [2 pts] What is the probability that a patient has the disease-carrying gene variation G given that they have disease B?

b. [3 pts] Consider the following Bayes Net, where each variable can take two possible values.



(i) [1 pt] What is the size of this Bayes Net?

(ii) [2 pts] You are given the query P (C|F), which you would like to answer using variable elimination. What is the variable elimination ordering where the largest intermediate factor is created during variable elimination is as small as possible.

Elimination ordering: