CS 5384-001/-D01 – Logic for Computer Scientists
Homework 3 – Solution

- **Assigned: 11/17/25**
- **Due: 11/23/25, 12pm – scan and submit on Canvas**
- **Each problem carries 10 points for a total of 50 points.**
- **Please show all work.**
- **You are encouraged to share ideas, but please submit your original and unique work.**
- **If you have a question, please ask and do not make assumptions.**

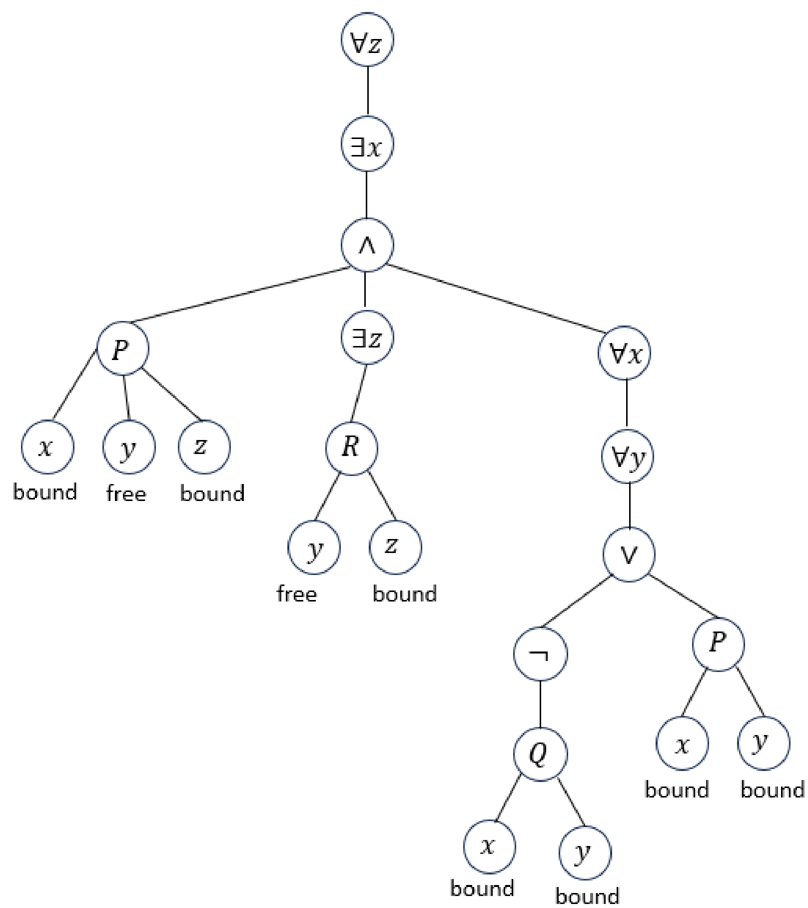
Problem 1: Consider

$$F = \forall z \exists x \left(P(x,y,z) \wedge \exists z\big(R(y,z)\big) \wedge \Big(\forall x \forall y\big(\neg Q(x,y) \vee P(x,y)\big)\Big)\right).$$

a. Draw a predicate logic tree of F. Determine the bound and free variables.
b. Show the scoping of all variables. You may use different colors as we did in class.

Solution:

a.



b.

$$F = \forall z \exists x \left(P(x,y,z) \wedge \exists z \big(R(y,z)\big) \wedge \Big(\forall x \forall y \big(\neg Q(x,y) \vee P(x,y)\big)\Big)\right)$$

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Problem 2: Write propositional statements for each of the following and use rules of inference to show the proof. Part (a) is solved as an example:

a. Polar bears live in the arctic and they rely on sea ice for hunting seals. Prove that polar bears rely on sea ice for hunting seals.

Solution:

p: Polar bears live in the arctic
q: Polar bears rely on sea ice for hunting seals

$$(p \land q) \to q$$

b. Joshua is an excellent runner. If Joshua is an excellent runner, then he can work as a running coach. Prove that Joshua can work as a running coach.
c. Jessica will work at a hair salon during summer. Prove that during the summer Jessica will work at a hair salon, or she will stay home.
d. The weather is over 100 degrees or there will be a kids baseball game. The temperature does not reach 100 degrees, prove that there will be a kids baseball game.

Solution:

b.

p: Joshua is an excellent runner
q: Joshua can work as a running coach
Use Modus Ponen

$$\big(p \land (p \to q)\big) \to q$$

c.

p: Jessica will work at a hair salon during summer
q: Jessica will stay home during summer
Use Addition

$$p \to (p \lor q)$$

d.

p: Weather is over 100 degrees
q: There will be a kids baseball game
Use Disjunctive Syllogism

$$\big(\neg p \land (p \lor q)\big) \to q$$

Problem 3: Consider the following:

"If it does not rain or if there is no thunder then the swimming classes will be held, and the lifesaving demonstrations will take place. If swimming classes are held, then students will learn a new swimming stroke. A new swimming stroke was not learned. This implies that it rained."

Write atomic proportions for information statements given above. Then express the statements above using propositional logic and use rules of inference (no tree or truth table) to prove that it rained. Please use logic and no text explanation.

Solution:

p: It rains
q: There is thunder
r: Swimming classes will be held
s: Lifesaving demonstrations will take place
t: Students will learn a new swimming stroke

1.	$(\neg p \vee \neg q) \rightarrow (r \wedge s)$	Premise
2.	$\neg t$	Premise
3.	$r \rightarrow t$	Premise
4.	$\neg r$	From 2 and 3 using Modus Tollens
5.	$\neg r \vee \neg s$	From 4 and Addition
6.	$\neg(r \wedge s)$	From 5 and De-Morgan's law
7.	$\neg(\neg p \vee \neg q)$	From 1 and 6 using Modus Tollens
8.	$(p \wedge q)$	From 7
9.	p	From 8 and Simplification

Problem 4: Mark plays golf and is happy or Mark is unhappy and he sleeps. Define a maximum of three atomic propositions and express the underlined statement in CNF.

Solution:

G: Mark plays golf

H: Mark is happy

S: Mark sleeps

$(G \wedge H) \vee (\neg H \wedge S)$

$= \neg\left(\neg\big((G \wedge H) \vee (\neg H \wedge S)\big)\right)$

$= \neg\big(\neg(G \wedge H) \wedge \neg(\neg H \wedge S)\big)$

$= \neg\big((\neg G \vee \neg H) \wedge (H \vee \neg S)\big)$

$= \neg\big((\neg G \wedge H) \vee (\neg G \wedge \neg S) \vee (\neg H \wedge \neg S)\big)$

$= \neg\big((\neg G \wedge H) \vee (\neg G \wedge \neg S) \wedge (\neg H \vee H) \vee (\neg H \wedge \neg S)\big)$

$= \neg\big((\neg G \wedge H) \vee (\neg G \wedge \neg S \wedge \neg H) \vee (\neg G \wedge \neg S \wedge H) \vee (\neg H \wedge \neg S)\big)$

$= \neg\big(\underbrace{(\neg G \wedge H) \vee (\neg G \wedge \neg S \wedge H)} \vee \underbrace{(\neg G \wedge \neg S \wedge \neg H) \vee (\neg H \wedge \neg S)}\big)$, after rearranging terms

Apply anti-distribution

$= \neg\big((\neg G \wedge H) \wedge (\top \vee \neg S) \vee (\neg H \wedge \neg S) \wedge (\top \vee \neg G)\big)$

$= \neg\big((\neg G \wedge H) \wedge \top \vee (\neg H \wedge \neg S) \wedge \top\big)$

$= \neg\big((\neg G \wedge H) \vee (\neg H \wedge \neg S)\big)$

$= \neg(\neg G \wedge H) \wedge \neg(\neg H \wedge \neg S)$

$= (G \vee \neg H) \wedge (H \vee S)$

Problem 5: Define appropriate propositional letters and express the following statements in predicate logic.

a. Every CS5384 student sleeps late on weekends.
b. CS5384 students who wake up early on weekdays stay fresh throughout the day.
c. Some CS5384 students who sleep late all week stays fresh throughout the day if they play tennis in the afternoon.
d. All CS5384 students sleep at 10pm every day.

Solution:

$LWE(x)$: Student sleep late on weekend
$UWD(x)$: Student wake up early on weekday
$LWD(x)$: Student sleep late on weekday
$F(x)$: Student remain fresh all day
$T(x)$: Student play tennis in the afternoon
$S(x)$: Student sleep at 10pm every day

a. $\forall x LWE(x)$
b. $\forall x (UWD(x) \rightarrow F(x))$
c. $\neg \forall x \left(\left(LWE(x) \land LWD(x) \land T(x) \right) \rightarrow F(x) \right)$
d. $\forall x S(x)$