Problem #1

Part 1 - If it rains today, then sun will shine tomorrow

Step 1: Declare Atoms

Let α = It raing today
Let β = Sum will shine tomorrow

$$\alpha \rightarrow \beta$$

Part 2: When John and Mike meet tomorrow, they will go for movie or have cup of coffee together

Let γ = John and Mike meet tomorrow
Let δ = they will go for movie
Let ε = they will have coffee

$$\gamma \rightarrow (\delta \vee \varepsilon)$$

Problem #2 Prove

$$((X \vee Y) \wedge (X \to Z)) \to (Y \vee (Z \wedge \neg Y))$$

is a tautology

Lecture 18: Tautology by resolution reasoning

Simplify the consequent using absorption law or distribution

$Y \vee (Z \wedge \neg Y)$

- $= (Y \vee Z) \wedge Y \vee \neg Y$
- $= (Y \vee Z) \wedge T$
- $= Y \vee Z$

So our target becomes

$$((X \vee Y) \wedge (X \to Z)) \to Y \vee Z$$

Convert to conjunctive normal form

$$\neg (((X \vee Y) \wedge (X \to Z)) \to Y \vee Z)$$

Apply implication elimination

$$\neg (\neg ((X \vee Y) \wedge (X \to Z)) \vee (Y \vee Z))$$

Apply De Morgans law

$$(X \vee Y) \wedge (X \to Z) \wedge \neg (Y \vee Z)$$

Convert $(X \to Z)$ to $\neg X \vee Z$

$$(X \vee Y) \wedge (\neg X \vee Z) \wedge \neg (Y \vee Z)$$

Apply De Morgans to $\neg (Y \vee Z)$

$$(X \vee Y) \wedge (\neg X \vee Z) \wedge \neg Y \wedge \neg Z$$

CNF Clauses

1) $\{X, Y\}$, 2) $\{\neg X, Z\}$, 3) $\{\neg Y\}$, 4) $\{\neg Z\}$

Step 4 Resolution Refutation

resolve (1) and (3) $\{X, Y\}$ with $\{\neg Y\} \rightarrow \{X\}$

resolve (2) $\{X\}$ w/ $\{\neg X, Z\} \rightarrow \{Z\}$

resolve (4) $\{Z\}$ with $\{\neg Z)$ $\rightarrow$ empty clause

We derived the empty clause provin the negation is unsatisfiable therefore the original is a tautology

Problem #3
(Lecture 14) algabraic equivalance approach

I'll prove both directions to establish the biconditional

Direction 1: Prove $(\alpha \rightarrow (\beta \rightarrow \gamma)) \vdash (\beta \rightarrow (\alpha \rightarrow \gamma))$

Starting with the left side
Convert implications to disjunctions
$p \rightarrow q \equiv \neg p \vee q$

$\neg \alpha \vee (\beta \rightarrow \gamma)$
$\neg \alpha \vee (\neg \beta \vee \gamma)$

Apply associativity of $\vee$
$(\neg \alpha \vee \neg \beta) \vee \gamma$

Apply commutivity of $\vee$
$(\neg \beta \vee \neg \alpha) \vee \gamma$

Apply associativity backwards
$\neg \beta \vee (\neg \alpha \vee \gamma)$

Convert back to implications
$\beta \rightarrow (\neg \alpha \vee \gamma)$

Direction 2 Prove

$(\beta \to (\alpha \to y)) \vdash (\alpha \to (\beta \to y))$

Starting with

$\beta \to (\alpha \to y)$

1) Convert to disjunctions

2) Apply associativity

3.) Apply commutivity

4.) Apply associativity backwards

5.) Convert back to implications

$\alpha \to (\beta \to y)$

$\therefore$ Since both directions hold via valid equivalance laws (implication elimination associativity commutivity) we have proven.

$(\alpha \to (\beta \to y)) \equiv (\beta \to (\alpha \to y))$

<u>Problem #4</u> Truth table with Don't cares (Lecture 15 K-map SOP methodology)

Step: Write canonical SOP (sum of minterms)

$Row_0 = A'B'C'D$
$Row_3 = A'B'CD$
$Row_5 = A'BC'D$
$Row_7 = A'BCD'$
$Row_9 = AB'C'D$
$Row_{10} = AB'CD'$
$Row_{13} = ABC'D$

$\bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D}$
$A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}D$

Step 2, Derive POS from SOP using DeMorgans law

$\neg F = \neg (m_0 + m_3 + \dots + m_{15})$

$F_{pos} = \prod M(1,4,6,8,12,14)$

Part(c): Express F in POS form, then derive SOP

Step 1: write canonical POS
(product of maxterms)

$F = 0\,(1,4,6,8,12,14)$:

$$F_{pos} = (A+B+C+\bar{D})(A+\bar{B}+C+D)\cdot$$
$$(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)$$
$$(\bar{A}+\bar{B}+C+D)(\bar{A}+B+\bar{C}+D)$$

Step 2: Derive SOP from POS
Negate the POS apply De Morgans law

$$F_{sop} = \sum m(0,3,5,7,9,10,13,15) + d(2,11)$$

Part (d): Do the SOP forms match?

No, from (b) FOP uses only explicit 1's

From (c) deriving SOP from POS implicitly treats don't cares as helping optimize

However both are logically equivalant for the specified inputs (when F is defined)

Part (e) Do the PoS forms match? Yes they match

From (b) POS derived from SOP covers exactly the zeros

From (c) POS from the table covers the same zeros

Problem 5: K map with don't cares   Lecture 17

$F(A,B,C,D) = m(0,1,2,3,4,5,6,7,13,15) + d(14)$

Part 1: find F_{sop} directly from K-map

AB \ CD	00	01	11	10
00	1	1	1	1
01				
11				
10				

Problem 5 $F(A,B,C,D) =$

$M(0\ 1\ 2\ 3\ 4, 5, 6, 7, 13, 15) + d(14)$

	A	B	C	D	F
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	X
15	1	1	1	1	1

$F_{SOP} =$

	00	01	11	10	
00	1	1	1	1	0 1 3 2
01	1	1	1	1	4 5 7 6
11	0	1	1	X	12 13 15 14
10	0	0	0	0	8 9 11 10

SOP	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	1	1
AB	0	1	1	X
$A\bar{B}$	0	0	0	0

$F_{sop}\ \bar{A} + BD$

POS	$\bar{C}+\bar{D}$	$\bar{C}+D$	$C+D$	$C+\bar{D}$
$\bar{A}+\bar{B}$	1	1	1	1
$\bar{A}+B$	1	1	1	1
$A+B$	0	1	1	X
$A+\bar{B}$	0	0	0	0

$(A+B+\bar{C}+\bar{D})(A+\bar{B})$

Redo →

	00 (C'D')	01 C'D	11 CD	10 CD'
00 A'B'	1	1	1	1
01 A'B	1	1	1	1
11 AB	0 m12	1	1	X
10 AB'	0 m8	0 m9	0 m11	0 m10

Variable		
A	Fixed at 1	A'
B	Fixed at 1	B'
C	Varies (0 to 1)	Eliminated
D	Fixed at 0	D

Maxterm 2: $\boxed{(A' + B' + D)}$

A	Fixed at 1	A'
B	Fixed at 0	B'
C, D	Vary	eliminated

this group covers m8, m9, m10 m, 11

Maxterm 1 $\boxed{(\bar{A} + B)}$

$F_{pos} = (\bar{A} + B)(A' + B' + D)$

Prove $F_{SOP} = F_{POS}$

using propositional Theorems

$F_{POS} = (A+\overline{B})(A+C)$

Apply distributive law

$A + \overline{B}C$

We need to show this equals $\overline{A}+BD$

Using De Morgans law

$\overline{F} = \overline{\overline{A} + BD} = \overline{\overline{A}} \cdot \overline{BD} = A(\overline{B+D}$

So $\overline{F} = A\overline{B} + A\overline{D}$

$F = \overline{\overline{F}} = \overline{A\overline{B} + A\overline{D}} = \overline{A\overline{B}} \cdot \overline{A\overline{D}}$

$= (\overline{A} + B)(\overline{A} + D) = \overline{A} + BD$

$\therefore$ QED